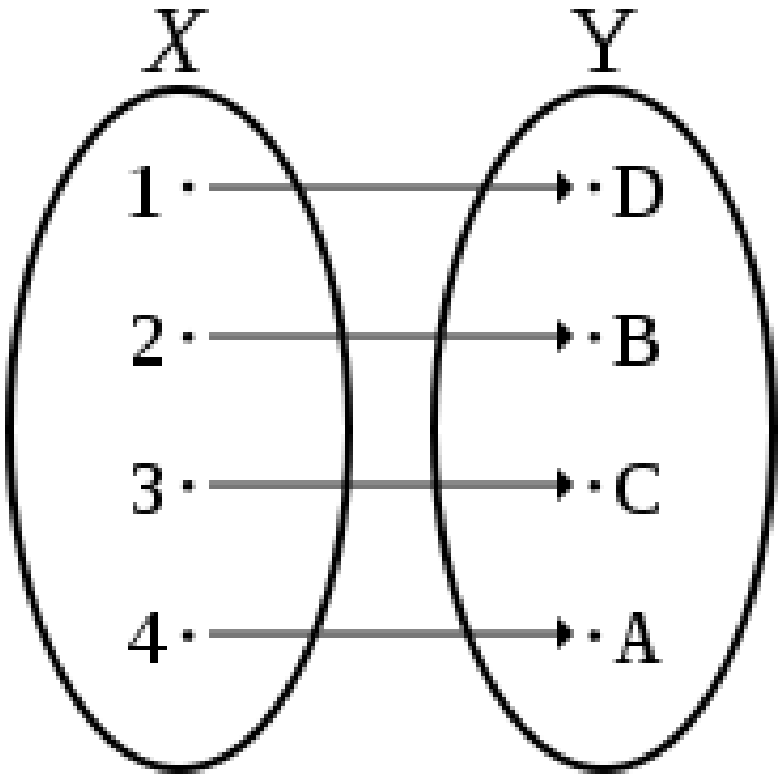
BIJECTIONS!

:D

(they’re more exciting than they sound)

Bijections?

- Given two sets, a bijection is a mapping that matches every element from one set to exactly one element in the second set.



Bijective Problems

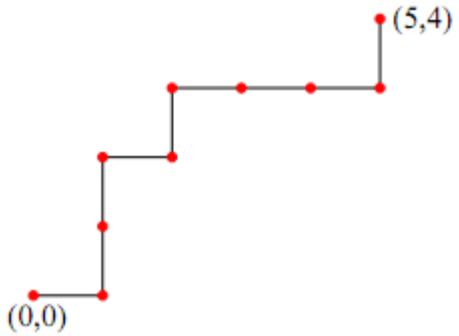
- Combinatorial problems can be solved by creating a bijection – that is, bijecting a difficult enumeration to a problem that we can enumerate.
- Example. Given a set of n elements, how many subsets are there?
- There are n elements.
- Each of the n elements is either in, or not in the subset.
- For each of the n elements there are 2 options.
- Therefore there are 2^n options in total, or 2^n subsets.

Some Rules

- Before we begin:
- No algebra.
- No induction.
- No recurrences.
- No generating functions.
- We can only use bijections.

Easy Problem

- Let $m, n > 0$. How many routes are there from $(0,0)$ to (m, n) if the allowed moves are $(0,1)$ and $(1,0)$?



- There are $m + n$ steps in total.
- m of those go up, and n of those go right.
- From the set $\{1,2, \dots, m + n\}$ choose m to go up, and the rest of them go right.
- Therefore there are $\binom{m+n}{m}$ routes.

Another Easy Problem

- Show that there are F_{n+1} ways to tile a $2 \times n$ board with dominoes.

- Two options, either we are left with $2 \times (n-1)$ or $2 \times (n-2)$.
- This defines the Fibonacci numbers.

Final Easy Problem

- A composition of order n is a sequence of positive integers that sum up to n.
- Show that the number of compositions of order n is 2^{n-1}.

- Let n balls be arranged in a row.
- There are $n - 1$ spaces between the balls.
- In any subset of the $n - 1$ spaces we may erect a barrier.
- Each forms a unique composition.
- There are 2^{n-1} such subsets.