COMC 2010 Unofficial Solutions

Henry Wise Wood Math Club 11/29/2010

1. Find $\frac{(9+5)^2-(9-5)^2}{9\times5}$

2. Solve $x - (8 - x) = 8 - (x - 8)$

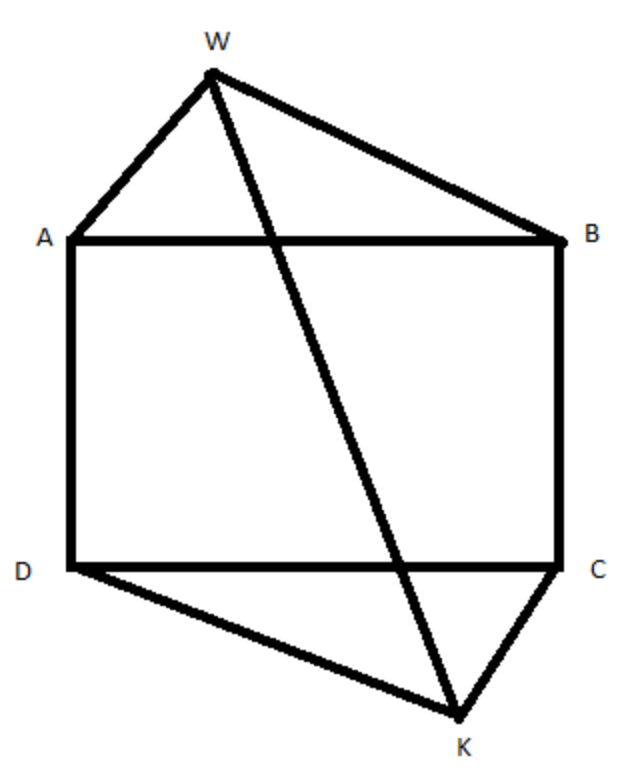
3. Three circles centered at O, CD passes through B, A, O. OA=2, OB=4, OC=6, then what is the area of the shaded region?

4. How many digits in $\frac{(3.1\times10^{7})(8\times10^{8})}{2\times10^{3}}$

5. What point on $y = x$ is closest to $P(-3,9)$?

6. On a exam, the average of students who studied was 90%, the average of students who did not study was 40%, and the class average was 85%. What percentage of the class did not study?

7. ABCD is a rectangle, AB=20, BC=10, WA=KC=12, WB=KD=16, find WK.



8. Solve $(x^2 + 3x + 2)(x^2 - 2x - 1)(x^2 - 7x + 12) + 24 = 0$

1a. Find C

A	A	50
B	C	44
37	57	

1b. Find n

D	D	D	30
F	F	E	55
F	E	E	50
50	n	40	

1c. Find P+Q

P	Q	T	R	20
Q	P	T	R	20
R	R	R	T	33
T	T	T	R	19
20	20	19	33	

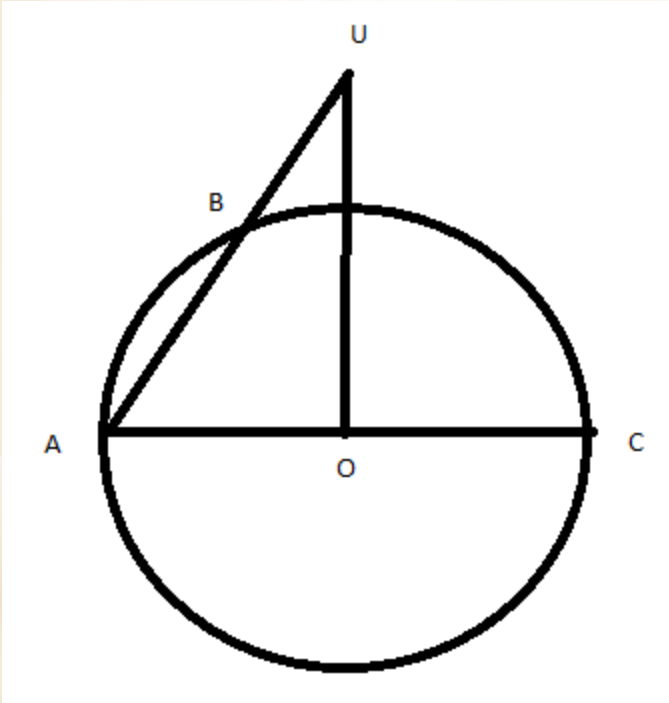
2a. Parabola $y = x^2 - 4x + 12$ intersects line $y = -2x + 20$ at A and B. Find the coordinates of A and B.

2b. Find the midpoint M of AB.

2c. A line parallel to $y = -2x + 20$ intersects the parabola at $P(p, p^2 - 4p + 12)$ and $Q(q, q^2 - 4q + 12)$. Prove $p + q = 2$.

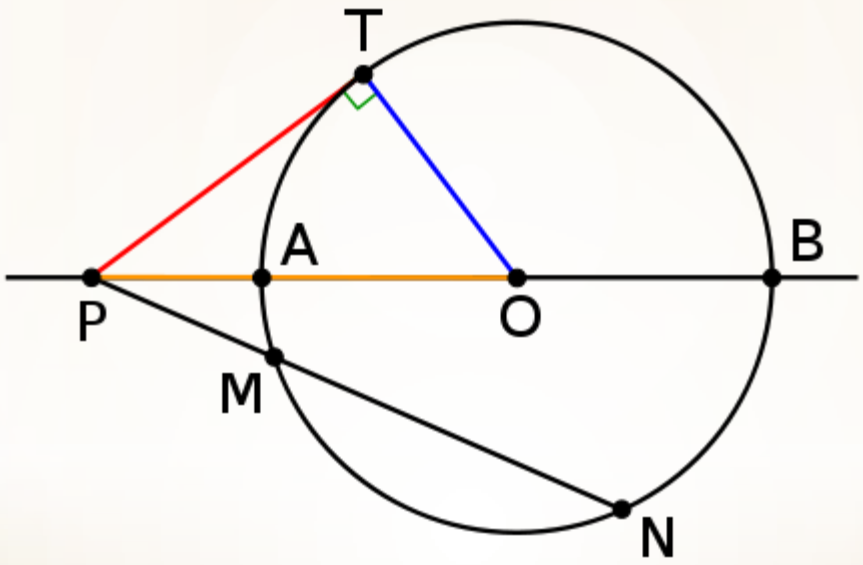
2d. N is the midpoint of PQ. Explain why MN is vertical.

3a. O is the center of the circle with diameter AC, radius 1. B is a point on the circle and AB is extended to P with BP=1. Let S be the set of points P. If U is in S and UO is perpendicular to AC, find UO.



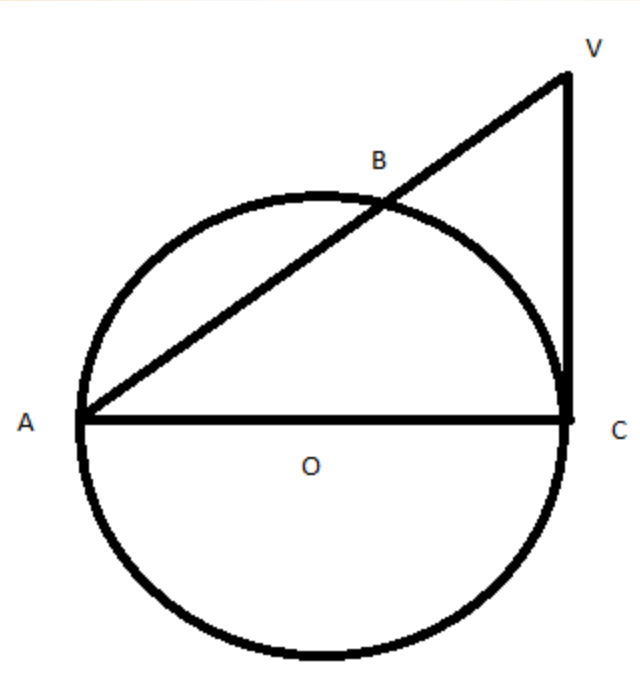
3b. V is in S and VC is perpendicular to AC. Find VC.

Power of a Point Theorem



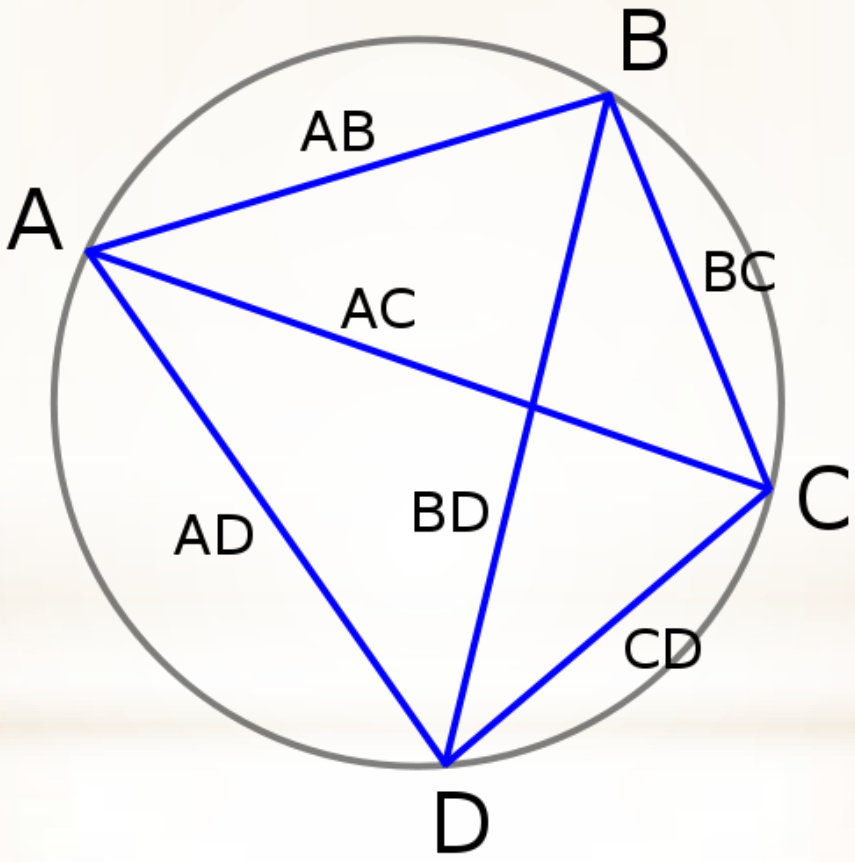
$$PT^2 = PM \cdot PN$$

3b. V is in S and VC is perpendicular to AC. Find VC.



3c. Do all points in S lie on one circle?

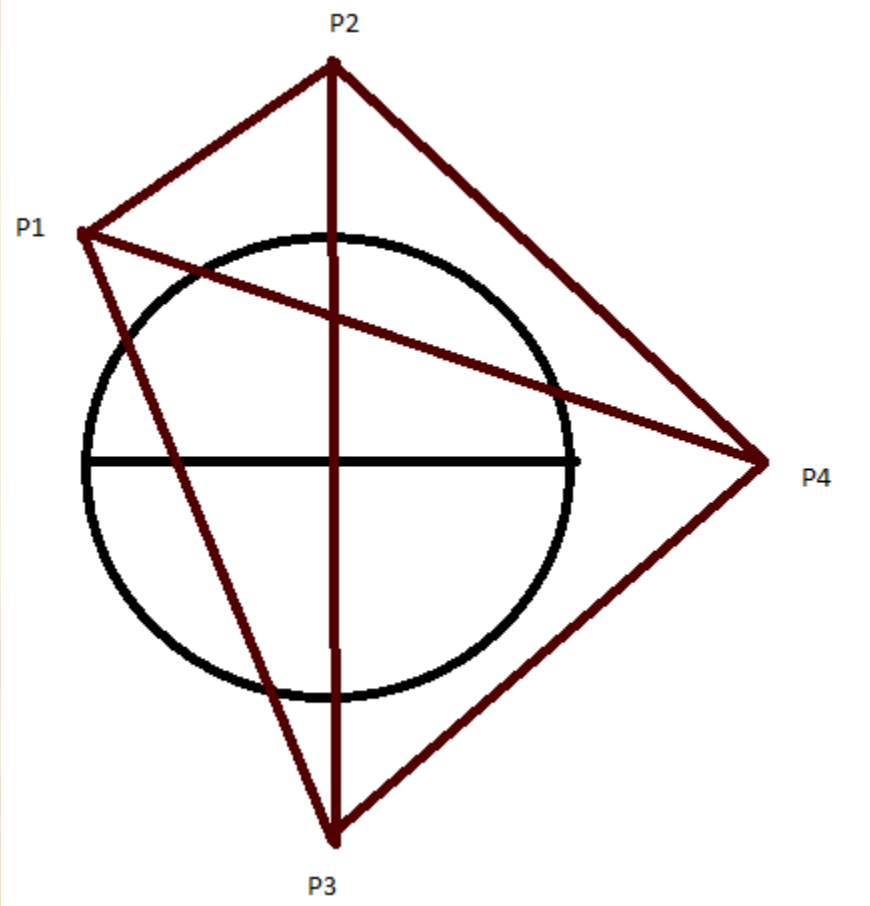
Ptolemy's Theorem



$$(AB)(CD) + (AD)(BC) = (AC)(BD)$$

3c. Do all points in S lie on one circle?

NO.



4a. $f(x) = \left(x + \frac{1}{x}\right) - \left\lfloor x + \frac{1}{x} \right\rfloor, x > 0$. Determine all x such that $f(x) = x$.

4b. Suppose that $x = \frac{a}{a+1}$ for some integer $a > 1$. Prove $x \neq f(x)$ but $f(x) = f\big(f(x)\big)$.

4c. Prove that there are infinitely many rational numbers u, $0 < u < 1$, so that $u, f(u), f\big(f(u)\big)$ are all distinct and $f\big(f(u)\big) = f\left(f\big(f(u)\big)\right)$.

I don't know how to do this problem either :(