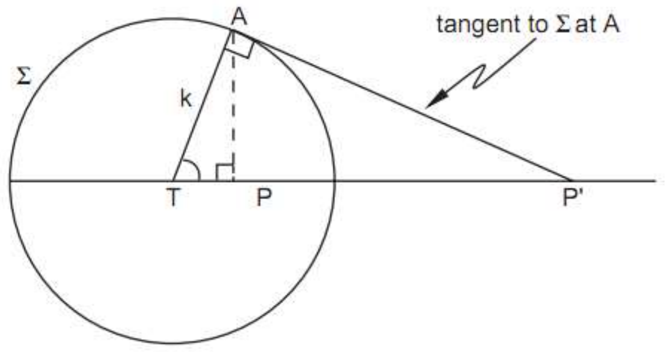
INVERSIVE GEOMETRY

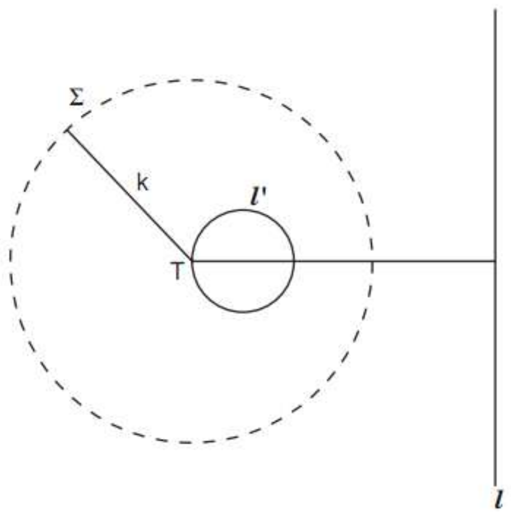
MATH CLUB 2/14/2011

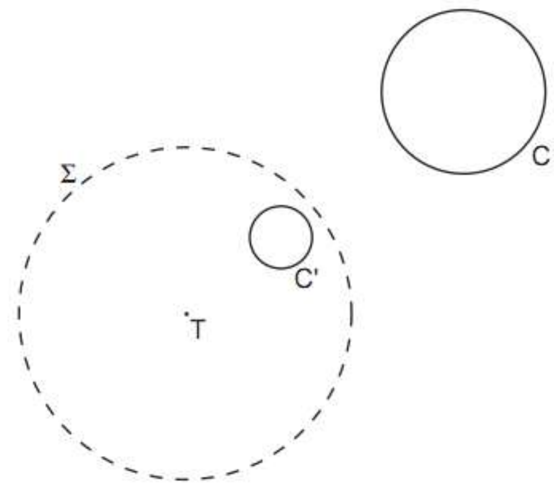
THE INVERSION



- The inversion is a transformation about a circle of radius k.
- Basically the inversion turns the space inside out, so everything inside the circle goes outside and everything outside goes inside.
- The closer a point is to the center, the farther away the inverse is.
- The inverse of a point P is the point P' such that $\frac{TP}{k} = \frac{k}{TP'}$, or $TP \cdot TP' = k^2$.

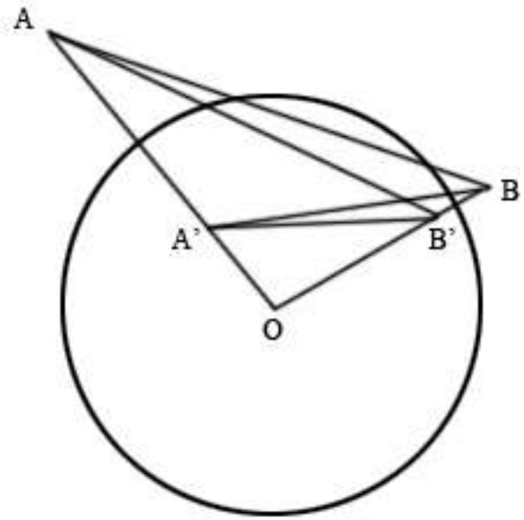
BASIC PROPERTIES OF THE INVERSION





- A line **passing through** T inverts into itself.
- A line **not passing through** T inverts into a circle **passing through** T.
- A circle **passing through** T inverts into a line **not passing through** T.
- A circle **not passing through** T inverts into another circle.
- I'm not going to prove all of these here.
- Basically an inversion turns lines into circles and circles into lines.

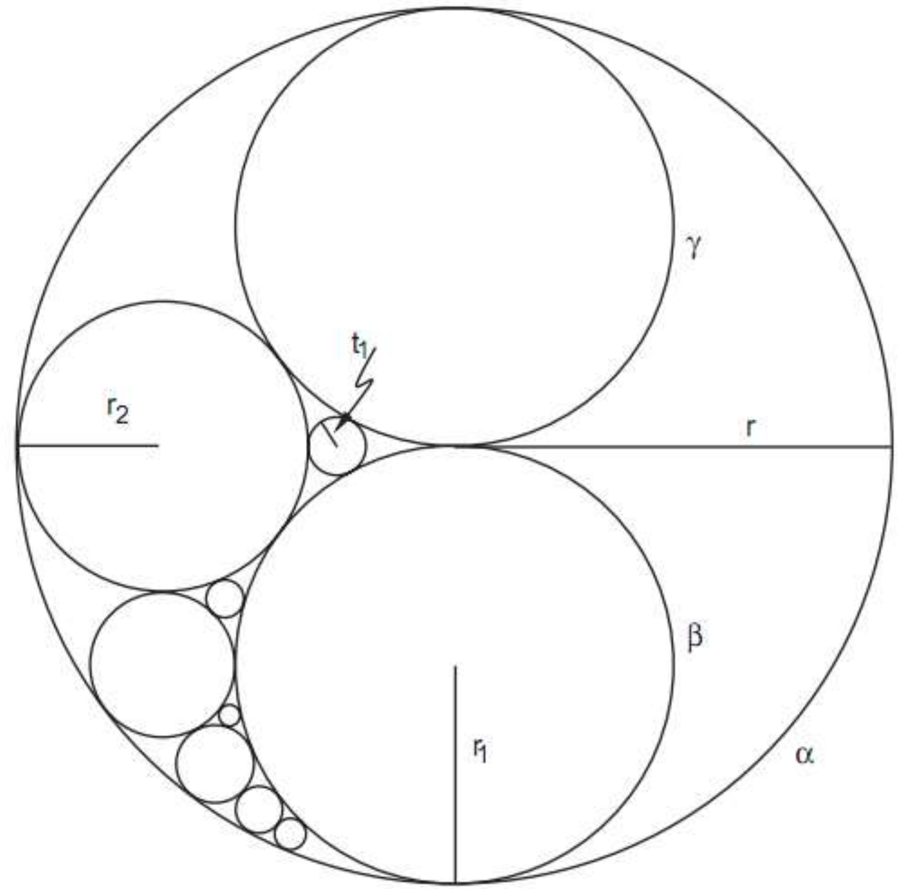
INVERSION AND SIMILAR TRIANGLES



- An important property of the inversion is that angles are preserved (to an extent).
- Here $OA \cdot OA' = k^2$ and $OB \cdot OB' = k^2$ so $\frac{OA}{OB} = \frac{OB'}{OA'}$.
- Thus triangles OAB and $OB'A'$ are similar. We have $\angle OAB = \angle OB'A'$.
- The ratio of similarity, or $\frac{OB'}{OA}$ is $\frac{k^2}{OB \cdot OA}$. Thus $A'B' = \frac{k^2 AB}{OA \cdot OB}$.

REALLY EASY PROBLEM #1

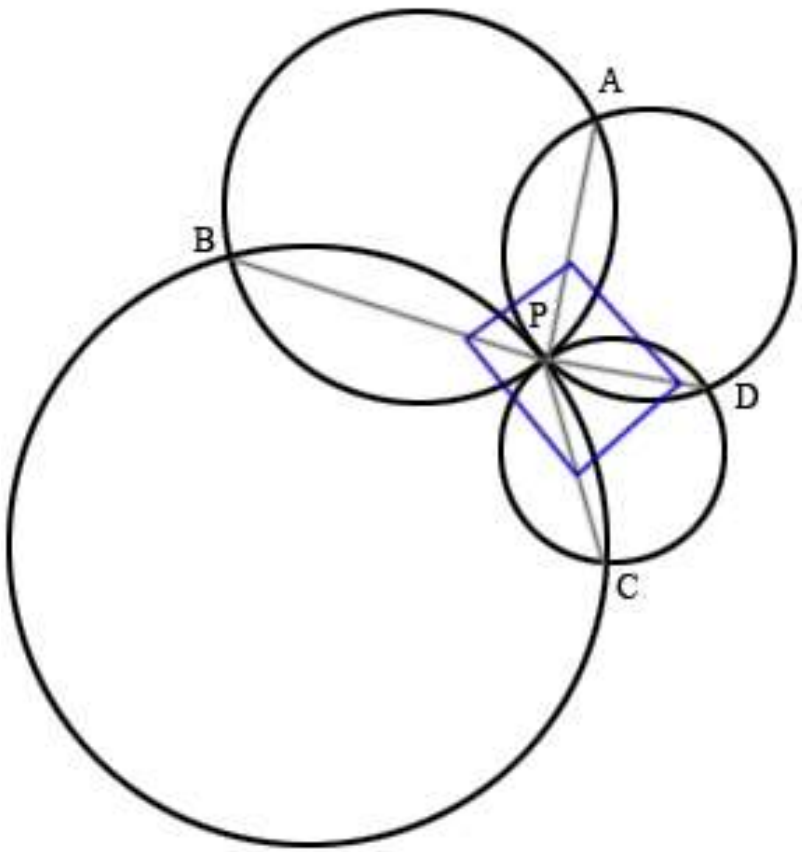
Proposed by Hotta Jinsuke (Japan), 1788



REALLY EASY PROBLEM #2

International Mathematical Olympiad 2003 Shortlist (G4)

Let Γ_1, Γ_2, Γ_3, Γ_4 be distinct circles such that Γ_1, Γ_3 are externally tangent at P, and Γ_2, Γ_4 are externally tangent at the same point P. Suppose the circles intersect at four distinct points A, B, C, D. Then prove that $\frac{AB \cdot BC}{AD \cdot DC} = \frac{PB^2}{PD^2}$.



REALLY EASY PROBLEM #3

Israeli Mathematical Olympiad 1995

PQ is the diameter of a semicircle. A circle is tangent to the semicircle and tangent to PQ at C. AB is perpendicular to PQ and is tangent to the circle; it intersects the semicircle at A and PQ at B. Prove that AC bisects $\angle PAB$.